

Holonomy group of the cone over a pseudo-Riemannian manifold

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Abstract

By a classical theorem of Gallot (1979), a Riemannian cone over a complete Riemannian manifold is either flat or has irreducible holonomy. This theorem has important applications for description of Killing spinors on a Riemannian manifolds with a spinor structure.

We consider metric cones with reducible holonomy over pseudo-Riemannian manifolds. First we describe the local structure of the base of the cone when the holonomy of the cone is decomposable. For instance, we find that the holonomy algebra of the base is always the full pseudo-orthogonal Lie algebra. One of the global results is that a cone over a compact and complete pseudo-Riemannian manifold is either flat or has indecomposable holonomy. Then we analyse the case when the cone has indecomposable but reducible holonomy, which means that it admits a parallel isotropic distribution. This analysis is carried out, first in the case where the cone admits two complementary distributions and, second for Lorentzian cones. We show that the first case occurs precisely when the local geometry of the base manifold is para-Sasakian and that of the cone is para-Kählerian. For Lorentzian cones we get a complete description of the possible (local) holonomy algebras in terms of the metric of the base manifold.

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