The geometry of g-natural contact metric structures on the unit tangent sphere bundle

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Abstract

The unit tangent sphere bundle T_1M, equipped with its standard contact metric structure $(\eta, \bar{g})$, is among the most known and studied examples of contact metric manifolds. Geometric properties of $(T_1M, \eta, \bar{g})$ influence those of the base manifold M itself, and conversely. Several contact metric properties on T_1M turn out to correspond to very rigid properties for the base manifold M [3].

We replace the standard contact metric structure of T_1M by a three-parameter family of contact metric structures $(\tilde{\eta}, \tilde{G})$, whose Riemannian metrics $\tilde{G}$ are g-natural. A g-natural metric G on the tangent bundle TM is the image associated to the Riemannian metric g of M by a first order natural operator [4]. A g-natural metric on T_1M is a metric $\tilde{G}$ induced by G on the unit tangent sphere bundle. Classic examples of Riemannian metrics on T_1M, like the Sasaki metric, the Cheeger-Gromoll metric and the metric $\bar{g}$ of the standard contact metric structure of T_1M, are all g-natural.

We study the geometry of $(T_1M, \tilde{\eta}, \tilde{G})$ and its relationship with geometric properties of the base manifold M. In particular, we give the necessary and sufficient conditions for a constructed contact metric structure: to be either Sasakian or K-contact; to satisfy some variational conditions; to define a strongly pseudo-convex CR-structure; to have either constant ξ-sectional curvature or constant φ-sectional curvature; to be a (k, μ)-space; to be locally symmetric.

The obtained results generalize classical theorems on the standard contact metric structure of T_1M and provide broad classes of examples for all the contact metric conditions above.

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