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Title: *Dupin Hypersurfaces With Four Principal Curvatures*

Abstract: A hypersurface M in the sphere S^n is said to be *proper Dupin* if the number g of distinct principal curvatures is constant on M and each distinct principal curvature is constant along each leaf of its corresponding principal foliation. Thorbergsson showed that the number g of distinct principal curvatures of a compact proper Dupin hypersurface M in S^n must be $1, 2, 3, 4$ or 6, the same as Münzner's restriction on the number of distinct principal curvatures of an isoparametric (constant principal curvatures) hypersurface in S^n. In the cases $g = 1, 2, 3$, compact proper Dupin hypersurfaces in S^n have been completely classified, and they are all equivalent to isoparametric hypersurfaces under Lie sphere transformations.

The cases of compact proper Dupin hypersurfaces with $g = 4$ or 6 principal curvatures have not yet been classified, although the multiplicities of the principal curvatures must be the same as for an isoparametric hypersurface. In the case $g = 4$, the multiplicities must come in pairs, and the principal curvatures can be ordered in such a way that $m_1 = m_2$ and $m_3 = m_4$. Further, a necessary condition for a Dupin hypersurface to be Lie equivalent to an isoparametric hypersurface is that it have constant Lie curvature (cross-ratio of the principal curvatures).

In this paper, we show that a compact proper Dupin hypersurface with $g = 4$ principal curvatures with multiplicities satisfying $m_1 = m_2 \geq 1$ and $m_3 = m_4 = 1$, and constant Lie curvature must be Lie equivalent to an isoparametric hypersurface in S^n. It remains an open question whether the conclusion holds without assuming that one pair of multiplicities is equal to one, although we conjecture that to be the case.

We also prove the following local result which is important in obtaining the global result above: if M is an irreducible connected proper Dupin hypersurface in S^n with four distinct principal curvatures with multiplicities $m_1 = m_2 \geq 1, m_3 = m_4 = 1$, and constant Lie curvature $r = -1$, then M must be Lie equivalent to an isoparametric hypersurface in S^n.