

Ricci Curvature, Hausdorff Distance and Almost Solvability of Fundamental Groups

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A fundamental question in Riemannian geometry is to investigate the relationships between curvature and topology of a Riemannian manifold. The classical Myers' theorem shows that the fundamental group of a compact Riemannian manifold with positive Ricci curvature must be finite. It is interesting to investigate the problem that "What can be said about the fundamental group of a compact positively Ricci curved manifold depending only on the dimension of the manifold, except that it is finite?" In this short lecture, we will give some new information on the topology of compact Riemannian manifolds with positive Ricci curvature. Sharp estimates on the first betti number of these manifolds and the ratio of diameters between the universal covering of such a manifold and the manifold itself will be given. Moreover, a weaker Margulis lemma was also obtained for Riemannian manifolds with a lower Ricci curvature bound. Our main method is the theory of equivariant Hausdorff distance developed by Fukaya and Yamaguchi. Some interesting examples about this result will be discussed.

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