

Global properties of indefinite metrics with parallel Weyl tensor

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Abstract

A pseudo-Riemannian manifold (M, g) of dimension $n > 3$ which is neither conformally flat nor locally symmetric, such that the Weyl conformal tensor of g is parallel, is called an ECS manifold (short for “essentially conformally symmetric”). All ECS metrics are indefinite, and numerous examples of open ECS manifolds are known. We prove that compact ECS manifolds exist in all dimensions $n > 4$ which have the form $n = 3j + 2$, and in such dimensions they realize all indefinite metric signatures. The ECS manifolds shown to exist are nontrivial torus bundles over the circle, and arise from a construction that a priori yields bundles over the circle, having as the fibre either a torus, or a 2-step nilmanifold with a complete flat torsionfree connection; our argument only realizes the torus case. Various particular details of the above existence theorem illustrate more general facts. Namely, every compact ECS manifold has an infinite fundamental group, its Euler characteristic is zero, and its real Pontryagin classes all vanish; for any compact ECS Lorentzian manifold (M, g) , some two-fold covering manifold of M is a bundle over the circle; and, finally, every four-dimensional ECS Lorentzian manifold is noncompact.