On 6-dimensional pseudo-Riemannian almost g.o. spaces

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Homogeneous geodesics on homogeneous Riemannian manifolds were studied for example in [8], [10]. In physics, Penrose limits along null homogeneous geodesics are studied in [5] and [11]. In [11], it is shown that the Penrose limit of a Lorentzian spacetime along a homogeneous geodesic is a homogeneous plane wave and the Penrose limit of a reductive homogeneous spacetime along a homogeneous geodesic is a reductive homogeneous plane wave. In mathematics, the first results for homogeneous geodesics on pseudo-Riemannian homogeneous manifolds were obtained in [1], [2] and [3].

The study of pseudo-Riemannian g.o. spaces (pseudo-Riemannian homogeneous spaces whose all geodesics are homogeneous) started with the paper [3]. In this paper, we considered 5-dimensional, 6-dimensional and 7-dimensional manifolds which were described with Riemannian metrics in [4], [6], [9] and geodesic graphs of these Riemannian g.o. manifolds were described in [4] and [7] (Kaplan's example of the first Riemannian g.o. manifold which is not naturally reductive was one of them). We modified the metrics and we obtained pseudo-Riemannian homogeneous spaces with *compact* isotropy group. We showed that these spaces are g.o. spaces and we described the discontinuities of geodesic graphs. At all points of discontinuity, there are different limits along different curves, but these limits are all finite.

In the present talk we show how to modify the Riemannian metrics on the 6-dimensional Kaplan's g.o. manifold and obtain homogeneous pseudo-Riemannian manifolds with *noncompact* isotropy group. These examples have the property that all geodesics are homogeneous up to a set of measure zero (and hence they are *not* g.o. spaces). We also show that the (incomplete) geodesic graphs are strongly discontinuous at the boundary, i.e., the limits along certain curves are always infinite.

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