# A geometric characterization from triple systems

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**Abstract**

Our aim is to give a charaterization of many mathematical (containing differential geometry) and physical fields by means of concept of triple systems (here, triple systems mean a vector space equipped with a triple product $< xyz >$).

In this note, we will give some examples of Peirce decomposition of generalized Jordan triple systems of second order. It seems that the concept of such triple sysems is useful in differential geometry as well as mathematical physics.

A $2\nu + 1$ graded Lie algebra is a Lie algebra of the form $g = \oplus_{k=-\nu}^{\nu} g_k$ such that $[g_k, g_l] \subset g_{k+l}$. Then it is well-known that 3-graded Lie algebras are essentially in bijection with certain theoretic objects called Jordan pairs. I.L.Kantor has remarked that more general graded Lie algebras correspond to generaliged Jordan triple systems. In particular, the graded Lie algebra

$$g_{-2} \oplus g_{-1} \oplus g_0 \oplus g_1 \oplus g_{2-2}$$

has a structure of a triple product on the subspace $g_{-1}$, that is, named a generalized Jordan triple system(GJTS) of second order or a (-1,1)-Freudenthal-Kantor triple system. And $g_{-1} \oplus g_1$ has a structure of a Lie triple system (corresponding to a symmetric space).

In [K-K], we have studied a Peirce decomposition of GJTS $U$ $(= g_{-1})$ of 2nd order. Roughly speaking, we have

$$U = U_{11} \oplus U_{13},$$

where for a tripotent element $e$ such that $eex = x$, $\forall x \in U$, we set $U_{11} = \{x \in U | xee = x\}$ and $U_{13} = \{x \in U | xee = 3x\}$.

We will discuss the corresponding geometrical object with these triple systems.

**References**

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