

DARBOUX TRANSFORMATION ON HARMONIC MAPS

K. LESCHKE

ABSTRACT. The classical Darboux transformation for isothermic surfaces can be generalized to conformal maps into the 4-sphere. These generalized Darboux transforms give a geometric interpretation of the spectral curve of a conformal 2-torus: Points on the spectral curve are exactly given by Darboux transforms. In the case of constant mean curvature surfaces we obtain a genuine generalization of the classical Darboux transformation which, however, preserves the constant mean curvature property up to Möbius equivalence. Evaluating this transformation on the harmonic Gauss map of the constant mean curvature surface gives a transformation on harmonic maps $N : M \rightarrow S^2$ which preserves harmonicity. We explain how this recipe may be used to define a transformation on harmonic maps in other situations, and discuss the spectral curve of a harmonic torus $N : T^2 \rightarrow S^2$ and the case when the harmonic maps is the conformal Gauss map of a Willmore surface.