Semiparallel submanifolds in space forms: classification problem for low dimensions

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According to the Publishing Agreement with Springer-Verlag (June 19, 2006) the author's monograph *Semiparallel Submanifolds in Space Forms: Envelopes of Symmetric Orbits* will appear in the near future. Here *semiparallel* means that the second fundamental form h satisfies the integrability condition $\bar{R}\circ h = 0$ of the differential system $\bar{\nabla}h = 0$ which characterizes the symmetric orbits. Our presentation deals with some results, not published yet, which are treated now in this monograph.

There is known the complete classification of semiparallel submanifolds M^m in Euclidean space for low dimensions $m \leq 3$: for $m = 2$ (surfaces) by J. Deprez in 1985, and for $m = 3$ by the author (partly with Riives) in 1990. In the monograph these results are extended to the case of space forms $N^n(c)$ with $c \neq 0$. For instance, the classification of semiparallel M^3 with principal codimension $m_1 =$dim span$h(X,Y) : X, Y \in T_xM^3$ is as follows.

If $m_1 = 1$ then such an M^3 is either

a) a hypersurface of rotation, whose profile curve has the natural equation $\kappa_g = cs(\sqrt{k - cs^2})^{-1}$, or

b) a parallel hypersurface, which is either spherical, or a product of a spherical surface $Sph^2(r)$ and a spherical curve.

If $m_1 = 2$ then such an M^3 is either

c) a Cartan variety (i.e. with orthogonal holonomic conjugate net of lines of curvature) thus with flat $\bar{\nabla}$, in an $N^5(c) \subset N^n(c)$, or

d) a warped product $B^1 \times_r Sph^2(1)$ with a nonconstant linear function r, or

e) a Segre submanifold $S_{(1,2)}(a)$ in a $S^5(a^2) \subset N^n(c)$, or a second order envelope of such submanifolds in $E^6 \subset N^7(c)$, $c < 0$ (so called logarithmic spiral tube), or

f) the product of a semiparallel surface with principal codimension 1 and a curve.

If $m_1 = 3$ then such an M^3 is

g) the product of a Veronese surface and plane curve of constant curvature in $N^6(c) \subset N^n(c)$, such that the curvature of this curve is connected to the quantity λ, which characterizes the Veronese surface, and is either $\sqrt{3c}\lambda(\sqrt{3\lambda^2 - c})^{-1}$, or $\sqrt{3}\lambda$ in the special case $c > 0$;

There exists no semiparallel M^3 with $m_1 = 4$ or $m_1 = 5$.

If $m_1 = 6$ then such an M^3 is

h) the second order envelope of three-dimensional Veronese submanifolds.