

# Darboux transformations: From elastic curves to elastic surfaces and beyond

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## Abstract

Reduction of the mNV flow of surfaces introduced by Konopelchenko and Taimanov in the 1990s to surfaces of revolution, or rather its meridian curves, yields the mKdV flow of curves. We will show that the classical Darboux transformation of isothermic surfaces applied to surfaces of revolution induces a Bäcklund transformation of the mKdV equation. The corresponding 1-soliton surfaces are then Darboux transforms of the round sphere, and consequently, as pointed out by Hertrich–Jeromin, Musso, and Nicolodi, they are rotationally symmetric Bryant surfaces (CMC 1 in hyperbolic space). We generalize the definition of soliton surfaces to compact surfaces in  $S^4$  so that it includes all Willmore spheres in  $S^4$ , minimal surfaces with planar ends in  $S^4$  minus a point, and all Bryant spheres. We show that using the generalized Darboux transformation introduced by Bohle, Leschke, Pedit, and Pinkall one gets all 1-soliton surfaces from twistor holomorphic surfaces in  $S^4$ .