

# Non-Euclidean method of the generalized geometry construction

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Conventionally a generalized geometry is constructed by the Euclidean method, when all propositions of the geometry are obtained from primordial axioms by means of logical reasonings and calculations. The main problem of this method is a test of consistency of original axioms. This consistency has been proved only for the proper Euclidean geometry. One suggests the alternative method of the geometry construction. It is based on the deformation of the Euclidean geometry, which has been constructed and whose consistency was proved. All propositions of the proper Euclidean geometry are listed and labelled by formulas in terms of the Euclidean world function  $\sigma_E$ . The set of formulas is powerful enough to label all propositions. The world function  $\sigma$  of the generalized geometry, given on the set  $\Omega$  of points, is defined by the relation  $\sigma = \frac{1}{2}d^2$ , where  $d$  is the distance and  $\sigma : \Omega \times \Omega \rightarrow \mathbb{R}$ ,  $\sigma(P, Q) = \sigma(Q, P)$ ,  $\sigma(P, P) = 0$ . It has been proved that labelling of all propositions of the Euclidean geometry in terms of the Euclidean world function  $\sigma_E$  is possible. After such a labelling all theorems of the Euclidean geometry turn into definitions. For instance, the cosine theorem turns into the definition of the scalar product  $(\mathbf{P}_0\mathbf{P}_1, \mathbf{P}_0\mathbf{P}_2) = \sigma_E(P_0, P_1) + \sigma_E(P_0, P_2) - \sigma_E(P_1, P_2)$  of two vectors  $\mathbf{P}_0\mathbf{P}_1$  and  $\mathbf{P}_0\mathbf{P}_2$ . The Gram's theorem on the necessary and sufficient condition of linear dependence of  $n$  vectors  $\mathbf{P}_0\mathbf{P}_1, \mathbf{P}_0\mathbf{P}_2, \dots, \mathbf{P}_0\mathbf{P}_n$  turns into the condition  $\det ||(\mathbf{P}_0\mathbf{P}_i, \mathbf{P}_0\mathbf{P}_k)|| = 0$ ,  $i, k = 1, 2, \dots, n$ , with the scalar product expressed via  $\sigma_E$ . The scalar product and the definition of the linear dependence do not refer to the concept of the linear space, and may be defined on any  $\sigma$ -space  $\{\Omega, \sigma\}$ . The same is valid for other propositions. To obtain the list of all propositions of the generalized geometry  $\mathcal{G}$ , described by the world function  $\sigma$ , it is sufficient to deform the Euclidean geometry  $\mathcal{G}_E$ , replacing  $\sigma_E$  by  $\sigma$  in the list of all propositions of the Euclidean geometry. The obtained set of generalized geometries is very powerful. It is not constrained by the topology of the set  $\Omega$ . What is especially important, it is not necessary to test the consistency of the constructed geometry, because this construction does not use any logical reasonings. Simplicity of the method and existence of multivariant non-Riemannian geometries admit one to construct such space-time geometries, where multivariant dynamics associated with the geometry is equivalent to the quantum description.